Computer-Based Representations and Manipulations of Paths in the Plane

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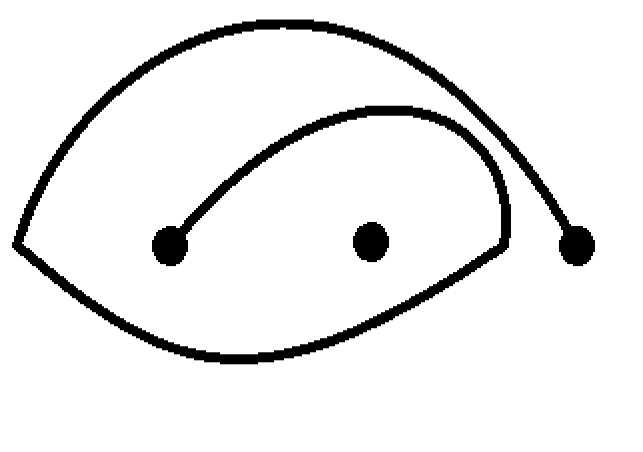
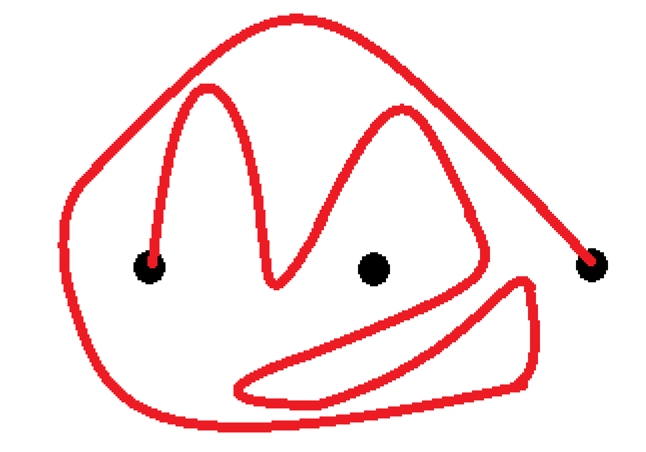
Background: Matching paths

What is a matching path?





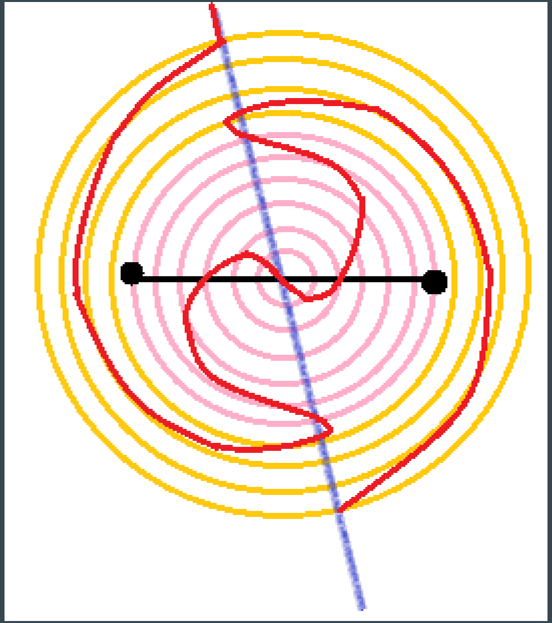
Motivation

Solution sets of polynomial equations in complex variables are interesting objects to study. There is a procedure to assign such a solution set an ordered collection of matching paths that capture its symplectic geometry. Conversely:

- An ordered collection of matching paths determine a symplectic manifold.
- Two different collections of paths can correspond to the same manifold.
- One operation we can perform on a collection of matching paths to obtain a new collection is called the Hurwitz move. The collection produced from the move corresponds to the same symplectic manifold.

Background: The Twist

The twist takes a matching path (such as the blue one) with respect to another path (in black) and produces another matching path (in red).

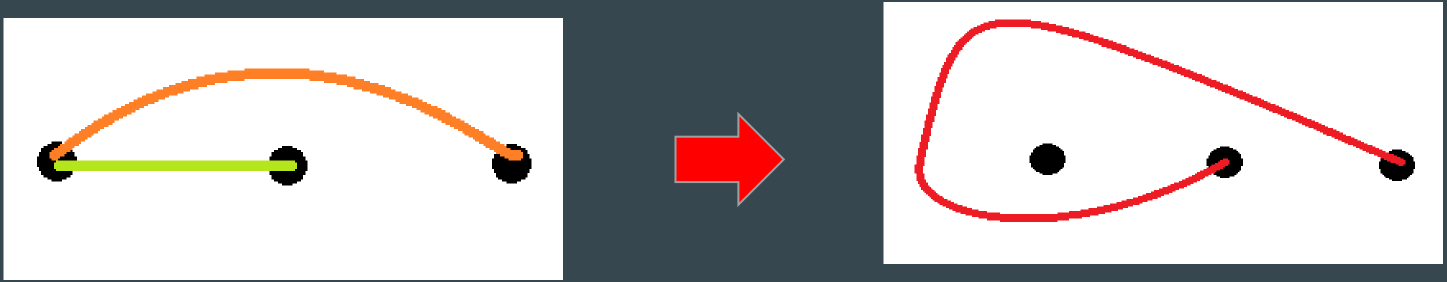


Procedure:

1. Consider the region around the matching path to respect to, and draw concentric circles in the region.
2. Starting from the innermost circle, rotate counterclockwise by a small angle. Increase the angle of rotation such that the angle is 180° at the circle around the marked points.
3. Gradually decrease the angle of rotation until the angle reaches 0° (the yellow region).

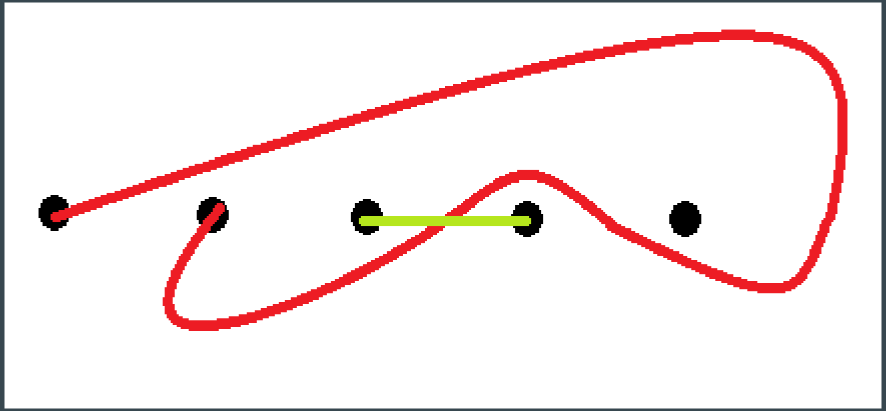
Background: The Twist, ctd

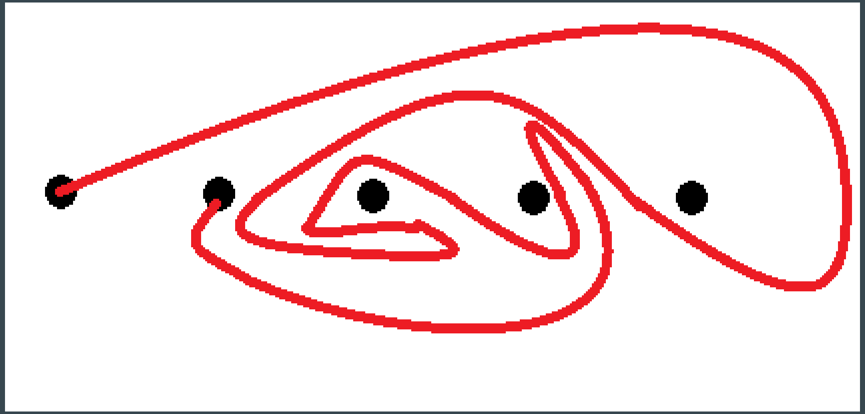
We can twist a path with respect to any other, it does not have to be a straight line connecting two adjacent points. The “straight line paths” are called **basic paths**.



Background: Simplifying Twists

The inverse of a twist is the result when a path has been rotated in the clockwise direction rather than the counter-clockwise direction.



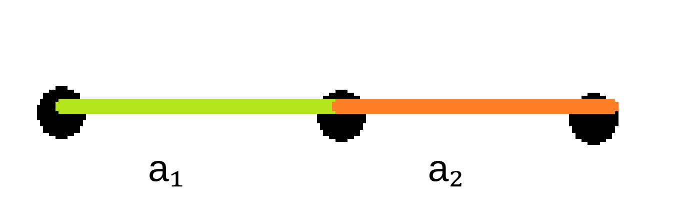


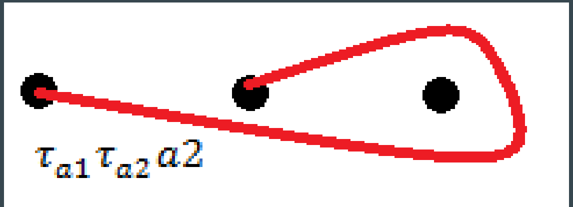
Twists can be “undone” in the sense that twisting with respect to a path, and then performing the inverse of the twist results in no change.

Background: Twist Words

The twist word is another way to classify a path. Basically, any matching path can be obtained from just one by performing twists with respect to basic paths.

Notation: τ denotes the twist. τ^{-1} denotes the inverse twist. τ_ab represents the twisting of a path b with respect to the path a.

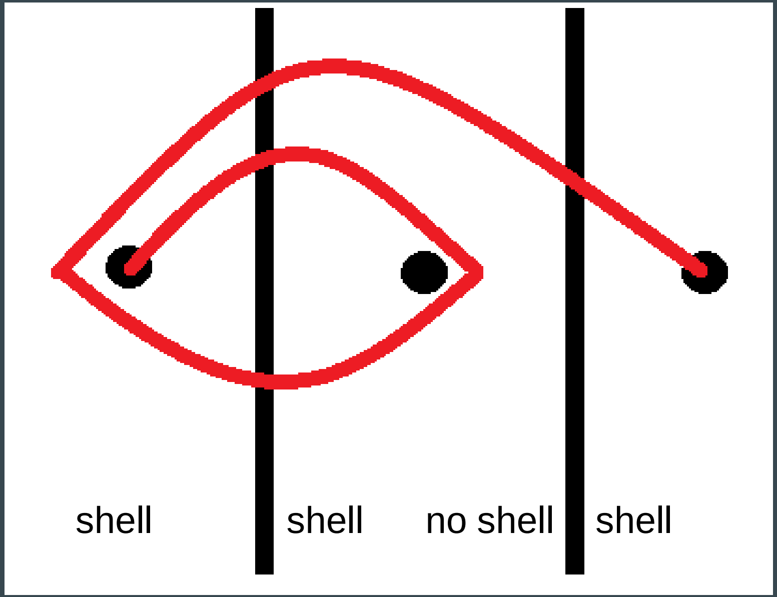




Background: Single Slit Data Representation

We can classify paths by using various sorts of representations. One way is **single slit data.**

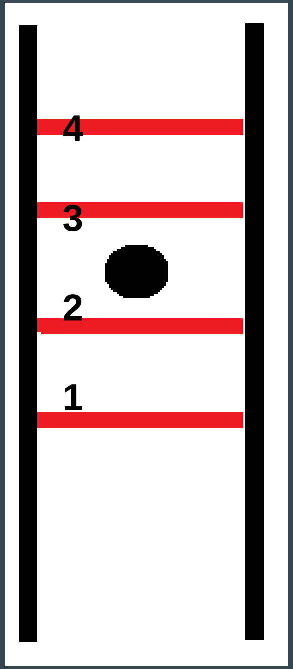
1. First draw vertical lines in between adjacent marked points
2. Deform your path such that it intersects **minimally** with the vertical lines
3. Find the number of times the path intersects each vertical line.
4. Determine whether there is a shell on each side of the vertical line.

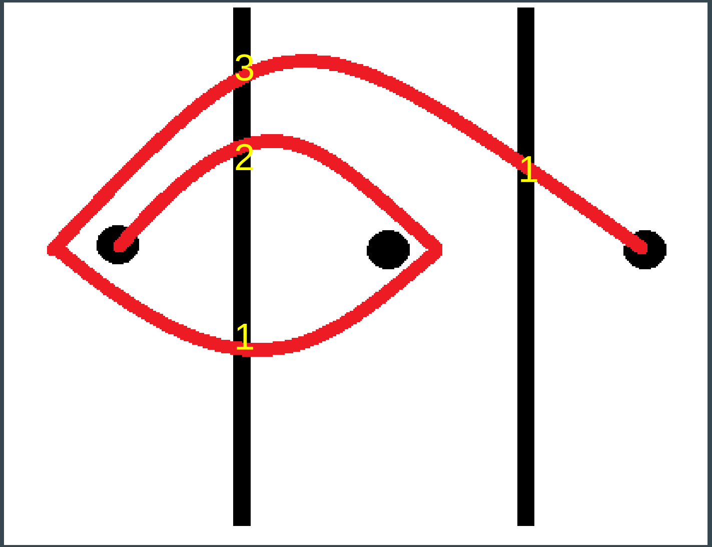


Background: Single Slit Data, ctd

5. Find the indices of the outermost shell

6. For slits with no shells, give the index of the uppermost line that goes below the marked point





Purpose of Project

Given two matching paths, we want to twist one path with respect to the other and return the result in the simplest possible form.

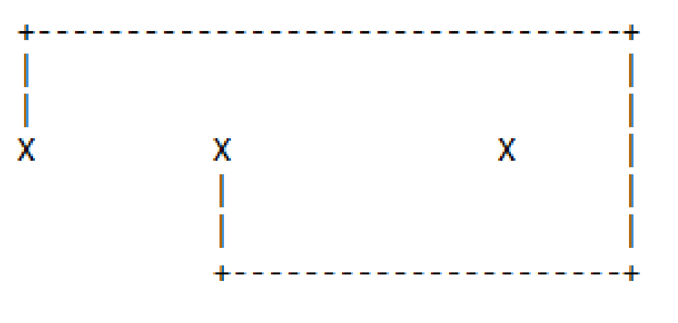
To do so, the following tasks are required:

1. Inputting the path to the computer
2. Finding the single slit data for the given path
3. Converting the single slit data to twist word data.
4. Performing the twist and obtaining the single slit data of the result.
5. Producing a diagram of the result.

Inputting Paths to the Computer

An “X” denotes a marked point.

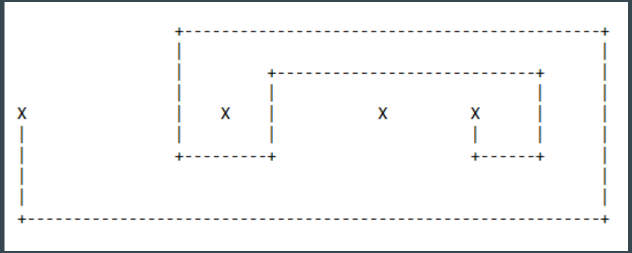
A “+” denotes a change of direction.

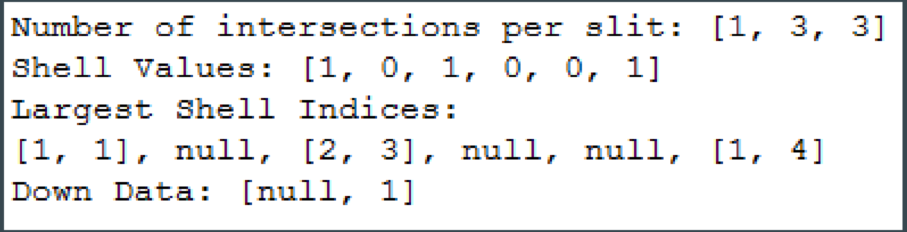


```
+---------------------------------+
|                                 |
|                                 |
X          X               X      |
           |                      |
           |                      |
           +----------------------+
```

From a Matching Path Diagram to Single Slit Data

I wrote a program that takes prompts the user for a .txt file of a matching path and outputs the single slit data. The program first scans the file and converts all the characters into a matrix. From there, it computes the rest of the single slit data.





```
Number of intersections per slit: [1, 3, 3]
Shell Values: [1, 0, 1, 0, 0, 1]
Largest Shell Indices:
[1, 1], null, [2, 3], null, null, [1, 4]
Down Data: [null, 1]
```

For the shell values, a 0 means there is no shell and a 1 means there is a shell.

Ongoing Research

I would like to continue by doing the following:

- Converting from single slit data to twist word data
- Making the program more user friendly by accepting more input formats.
- Searching for the simplest possible diagram for a given manifold (possibly using brute force algorithms)

Acknowledgements

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